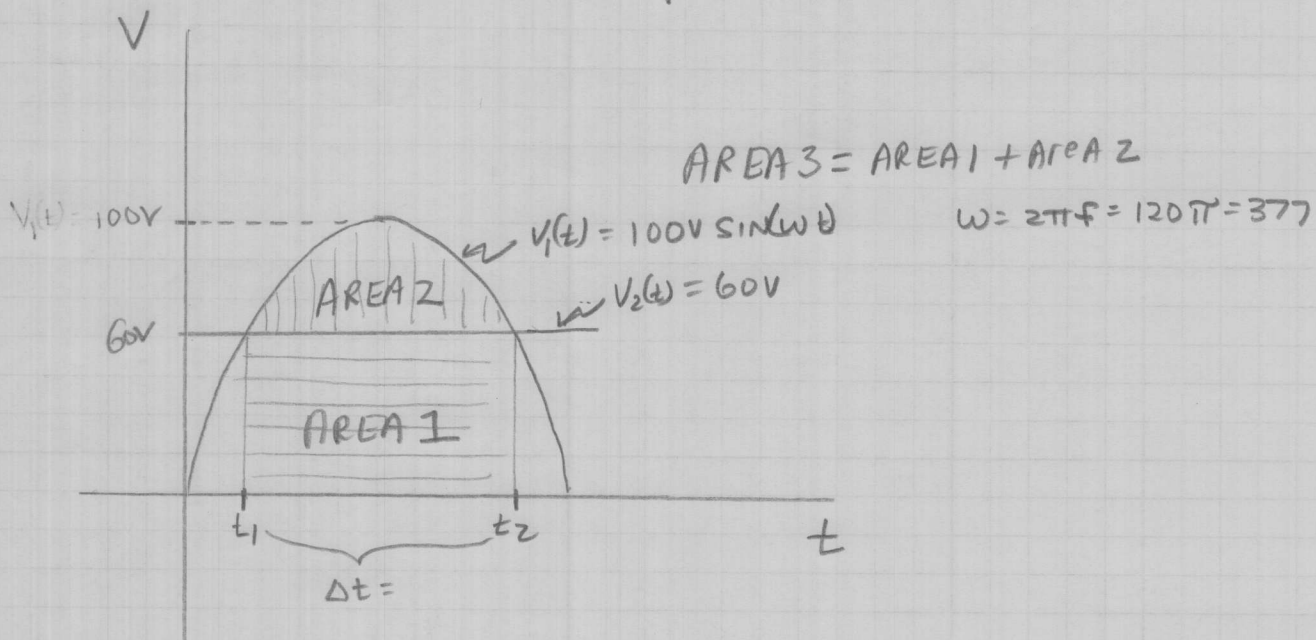


EXAMPLE 21-1

CALCULATION OF RMS VALUE  
OF PERIODIC WAVEFORM



$$V_1(t) = 100 \sin(\omega t) = 100 \sin(377t)$$

$$t_1 = \frac{1}{377} \sin^{-1}(0.6) = 1.707 \text{ ms}$$

$$t_2 = 8.33 \text{ ms} - 1.707 \text{ ms} = 6.626 \text{ ms}$$

$$\Delta t = t_2 - t_1 = 6.626 \text{ ms} - 1.707 \text{ ms} = 4.9195 \text{ ms}$$

$$AREA 1 = (60V)(4.9195 \text{ ms}) = 0.29517 \text{ V}\cdot\text{s}$$

$$\text{Area}_3 = V_{\text{RMS}} * \Delta t$$

$$V_{\text{RMS}} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t)^2 dt}$$

$$2\omega = 2(377) = 754$$

$$f(t) = 100 \sin(377t)$$

$$f^2(t) = 10^4 \sin^2(377t)$$

FROM LIST OF INTEGRAL OF TRIG FUNCTIONS

$$\int \sin^2 ax dx = \left[ \frac{x}{2} - \frac{1}{4a} \sin(2ax) \right] = \left[ \frac{x}{2} - \frac{1}{2a} \sin(ax) \cos(ax) \right]$$

$$V_{\text{RMS}} = \sqrt{\left( \frac{10^4}{4.9 \text{ms}} \right) \int_{t_1}^{t_2} \sin^2(377t) dt}$$

$$= \sqrt{\left( \frac{10^4}{4.9 \text{ms}} \right) \left[ \left( \frac{t_2}{2} - \frac{1}{754} \sin(377t_2) \cos(377t_2) \right) - \left( \frac{t_1}{2} - \frac{1}{754} \sin(377t_1) \cos(377t_1) \right) \right]}$$

$t_2 = 6.626 \text{ms}$                        $t_1 = 1.767 \text{ms}$

$$= \sqrt{\left( \frac{10^4}{4.9 \text{ms}} \right) \left[ \underbrace{(0.003313 + 0.0006366)}_{0.0039496} - \underbrace{(0.0008535 - 0.0006366)}_{0.0002169} \right]}$$

$2.03 \times 10^6$                        $0.0037327$

$$= \sqrt{(2.03 \times 10^6) (0.0037327)} = \sqrt{7587.6} = 87.1 \text{V}$$

$$\text{Area}_3 = V_{\text{RMS}} * \Delta t = 87.1 \text{V} * 4.9195 \text{ms} = 0.4285 \text{V} \cdot \text{s}$$

$$\text{Area 2} = \text{Area 3} - \text{Area 1}$$

$$\text{Area 2} = 0.4285 \text{ V}\cdot\text{s} - 0.29517 \text{ V}\cdot\text{s}$$

$$\text{Area 2} = 0.1333 \text{ V}\cdot\text{s}$$

$$I_{\text{max}} = \frac{\text{Area 2}}{L} = \frac{0.1333 \text{ V}\cdot\text{s}}{3.3 \text{ mH}} = \boxed{40 \text{ A}}$$

$$\int V_L dt = L \int I dt$$

$$\text{Area 2} = LI$$