SEMICONDUCTOR GENERATED WAVEFORM MODELING

And Waveform Harmonic Reduction by Filter Modeling.

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(Thanks to Mike Allin, McLaren Electronic Systems for his review)

With the proliferation of semiconductors in the power industry, waveform harmonic content and harmonic filter design have become issues of extreme interest. This paper describes a method of mathematically modeling three of the most common semiconductor generated waveforms. These are the pulse width modulated (PWM) waveform, the SCR phase controlled waveform, and the chopper generated waveform. Then we will show how to determine the harmonic coefficients and the total voltage and power harmonic distortion of the generated waveforms. Lastly, we will demonstrate a method of modeling components of a low pass filter used to reduce the harmonic coefficients and then determine the harmonic coefficients and total voltage and power harmonic distortion of the filtered waveform.

I CIRCUIT DESCRIPTION:

The first waveform that we will analyze is the pulse width modulated (PWM) waveform. This waveform is generated by several transistors configured in an H bridge[1]. The transistors are fed from a DC bus and the output of the bridge is the PWM waveform. See figure 1 for a schematic representation of a three phase H bridge circuit.

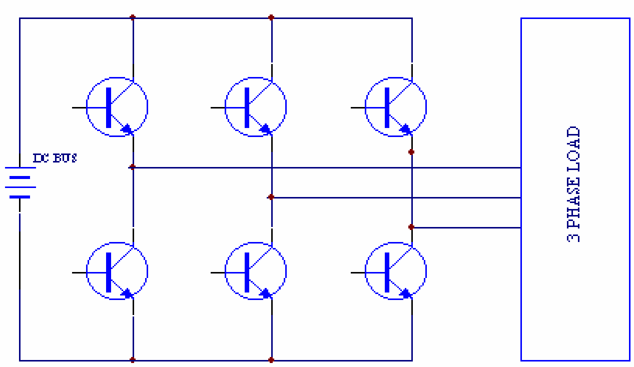

FIGURE #1: THREE PHASE H BRIDGE TRANSISTOR CONFIGURATION

In order to simplify analysis, we will look at the single line equivalent circuit as shown in figure #2. Note, that this assumes that the 3 phase load is balanced. If the three phase load is unbalanced, then symmetrical component analysis must be done to determine the circuit parameters. [2]

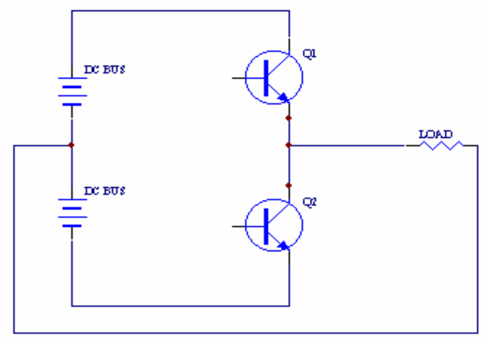

FIGURE #2: SINGLE LINE EQUIVALENT H BRIDGE CONFIGURATION:

The second waveform that we will model is the silicon controlled rectifier (SCR) controlled phase angle waveform. Unlike the transistor circuit shown above, the SCR device is placed directly in the AC circuit in series with the load[1]. The firing of the SCR is delayed by some time period. While the SCR is not gated, the output voltage is zero. When the SCR is gated, the output voltage is the line voltage from the time of gating to the next zero cross when the SCR commutates off. See figure #3 for an example of an SCR circuit.

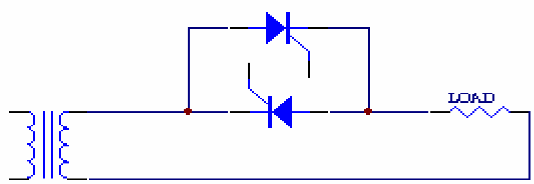

FIGURE #3: SCR CONTROLLED CIRCUIT

Lastly, we will model the waveform of a single transistor chopper circuit. This circuit is constructed of a transistor connected in series with the load[1]. The combination of these two components is connected across a DC bus. This circuit converts the DC voltage level of a DC bus to a chopper waveform by switching the transistor in series with the load on and off at a defined frequency and duty cycle. See figure #4 for an example of a chopper circuit.

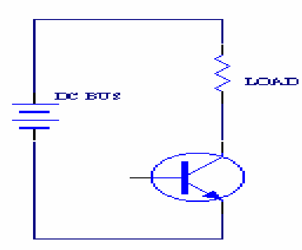

FIGURE #4 : TRANSISTOR CONTROLLED CHOPPER CIRCUIT.

II MODELING OF SEMICONDUCTOR GENERATED WAVEFORM

First, we will mathematically describe the pulse width modulated (PWM) waveform. Because we are using transistors, the output waveform is either at the positive of the dc bus or at the negative of the dc bus. To achieve a simulated sinusoidal waveform across the load, the transistors are switched in a pulse width modulated scheme. This means that the width of each pulse is modulated to achieve an approximately sinusoidal waveform across the load. Mathematically, this is achieved by modulating a sinusoidal waveform at the desired (base) frequency with a triangular waveform oscillating at some much higher (carrier) frequency. Our first step is to mathematically model these two waveforms.

First, the carrier waveform is simply a triangle waveform with period of Tc, and a peak of Vp. In discrete format, the waveform is evaluated as:

$$Fc(t) := \left(-Vp + \frac{4\cdot Vp}{Tc}\cdot t\right)\cdot\mu(t) + \frac{8\cdot Vp}{Tc}\cdot\left[\sum_{N=1}^{\infty}\left[(-1)^N\cdot\left(t - \frac{N\cdot Tc}{2}\right)\cdot\mu\left(t - \frac{N\cdot Tc}{2}\right)\right]\right]$$

It is a simple technique to model this waveform as two straight lines. The first half of this waveform can be modeled from time 0 to time Tc/2 as a straight line with a slope of 4*Vp/Tc and a Y intercept of -Vp. The second half of this waveform can be modeled from time Tc/2 to time Tc as a straight line with a slope of -4*Vp/Tc and a Y intercept of 3*Vp. For simplicity, we have normalized Vp to 1 and Tc to 1. This waveform generation is shown below and the waveform is graphically depicted in figure #5.

$K := 20$

$J1 := 1..K \qquad I1 := 1..\frac{K}{2} \qquad I2 := \frac{K}{2}+1, \frac{K}{2}+2..K \qquad TIME_{J1} := \frac{J1}{20}$

$V1_{I1} := 4\cdot\frac{I1}{K} - 1 \qquad V1_{I2} := -4\cdot\frac{I2}{K} + 3$

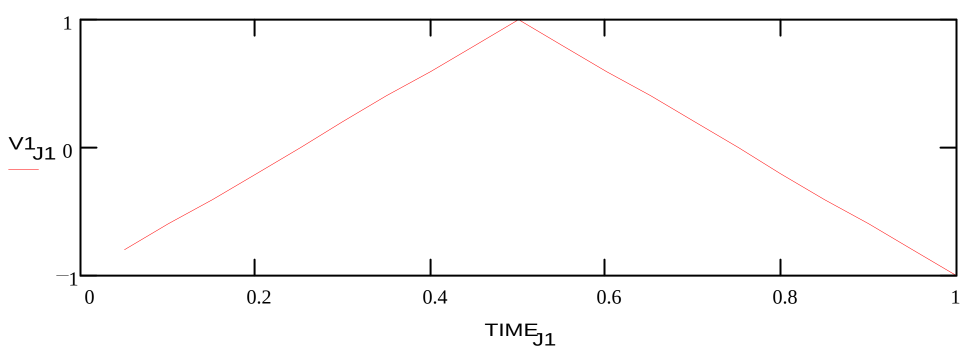

FIGURE #5: ONE PERIOD OF CARRIER WAVEFORM NORMALIZED TO 1 VOLT:

Now, we use iteration to generate multiple copies of the carrier waveform and displace each one from the proceeding one by a time period of Tc.

$I3 := K+1, K+2 .. \frac{3 \cdot K}{2}$ $\quad I4 := \frac{3 \cdot K}{2}+1, \frac{3 \cdot K}{2}+2 .. 2 \cdot K$

$I5 := 2 \cdot K+1, 2 \cdot K+2 .. \frac{5 \cdot K}{2}$ $\quad I6 := \frac{5 \cdot K}{2}+1, \frac{5 \cdot K}{2}+2 .. 3 \cdot K$

$I7 := 3 \cdot K+1, 3 \cdot K+2 .. \frac{7 \cdot K}{2}$ $\quad I8 := \frac{7 \cdot K}{2}+1, \frac{7 \cdot K}{2}+2 .. 4 \cdot K$

$I9 := 4 \cdot K+1, 4 \cdot K+2 .. \frac{9 \cdot K}{2}$ $\quad I10 := \frac{9 \cdot K}{2}+1, \frac{9 \cdot K}{2}+2 .. 5 \cdot K$

$V1_{I3} := V1_{I3-K}$ $\quad V1_{I4} := V1_{I4-K}$ $\quad V1_{I5} := V1_{I5-K}$ $\quad V1_{I6} := V1_{I6-K}$

$V1_{I7} := V1_{I7-K}$ $\quad V1_{I8} := V1_{I8-K}$ $\quad V1_{I9} := V1_{I9-K}$ $\quad V1_{I10} := V1_{I10-K}$

Now all that is left is to scale the above waveform to allow for the definition of peak waveform voltage (Vp) and time period (Tc). Since one of the standard electrical power frequencies is 60 Hz, we will initially define our modulated waveform at 60 Hz and our carrier frequency at 10 times the modulated frequency or 600Hz. Also, to simplify analysis, we will leave the peak voltage(Vp) at a value of 1. All these variables can be changed as required in order to simulate actual output waveforms. Ten cycles of our carrier waveform can be seen in figure #6.

$Vp := 1$ $\quad Fc := 600$ $\quad J10 := 1 .. 6400 \cdot K$ $\quad TIME_{J10} := \frac{J10}{Fc \cdot K}$

$J2 := 1 .. 10 \cdot K$ $\quad TIME_{J2} := \frac{J2}{Fc \cdot K}$ $\quad Vc_{J10} := Vp \cdot V1_{J10}$ $\quad TIME_K = 0.002$

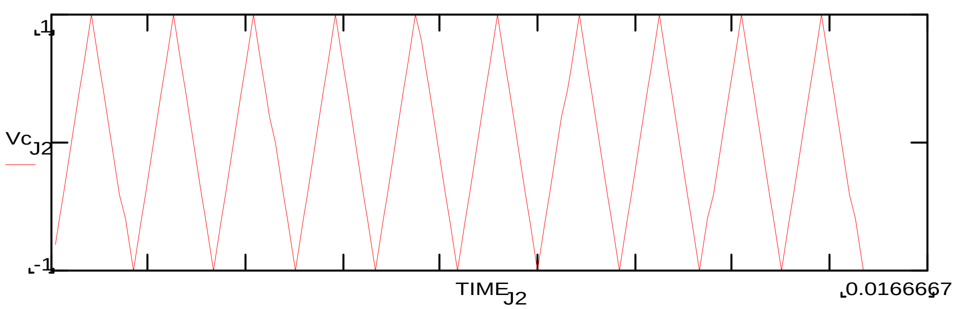

FIGURE #6: TEN PERIODS OF CARRIER WAVEFORM WITH Vp = 1 AND Fc = 600Hz:

Now we turn our attention to the actual base frequency that we want to modulate with our carrier frequency. As stated above, we will use a frequency of 60Hz as our base frequency and we will match our peak sine wave voltage with our peak carrier voltage. As a side note, in order to control the RMS value of the PWM waveform, the peak voltage of the base frequency can be varied. This will change the RMS value of the modulated waveform. In discrete format, the waveform is evaluated as:

$$Fb(t) := Vp \cdot \sin\left(\frac{2\cdot\pi}{Tb}\cdot t\right)\cdot\mu(t)$$

Using these values, we find our sine wave as shown below in figure #7.

$K2 := 100$

$T1 := K \cdot K2 \quad Fi := \frac{Fc}{K2} \quad Ti := \frac{1}{Fi}$

$J30 := 1 .. T1 \qquad Fi = 60$

$VSIN_{J30} := Vp \cdot \sin(2\cdot\pi\cdot Fi \cdot TIME_{J30}) \qquad T1 = 2 \cdot 10^3$

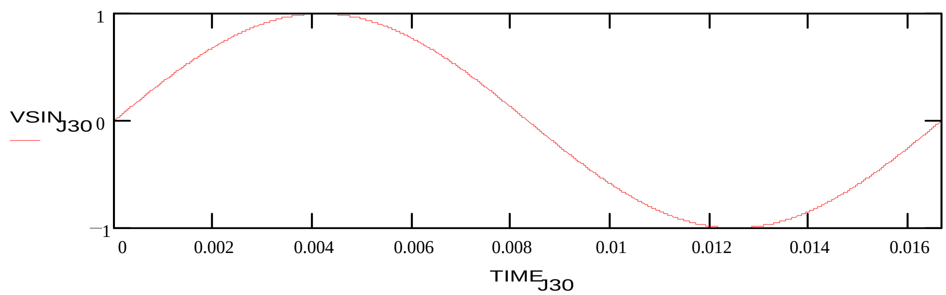

FIGURE #7: ONE PERIOD OF BASE WAVEFORM WITH Vp = 1 AND Fi = 60hz:

Now to mathematically generate the modulated waveform, we simply compare the carrier waveform to the base waveform at all instances in time. Whenever the base is greater than the carrier, we turn on one set of transistors, and whenever the carrier is greater than the base we turn on the opposing set of transistors. This is mathematically modeled as an if/then statement. If the base waveform is greater than the carrier waveform, then the modulated waveform is set equal to Vp. If the carrier waveform is greater than the base waveform, then the modulated waveform is sent equal to -Vp. For comparison purposes, the carrier and base waveforms are displayed below in figure #8 on the same graph. The resulting PWM waveform is shown in figure #9:

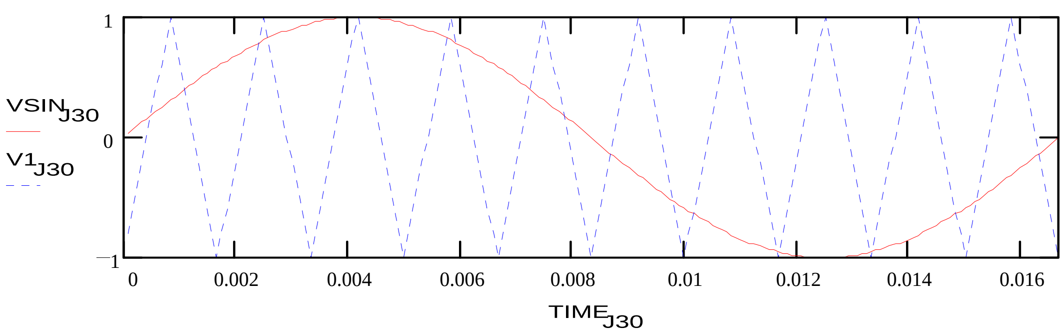

FIGURE #8: BASE AND CARRIER WAVEFORMS DISPLAYED ON ONE GRAPH

$PWM_{J30} := if\left(V1_{J30} < VSIN_{J30}, Vp, -Vp\right)$

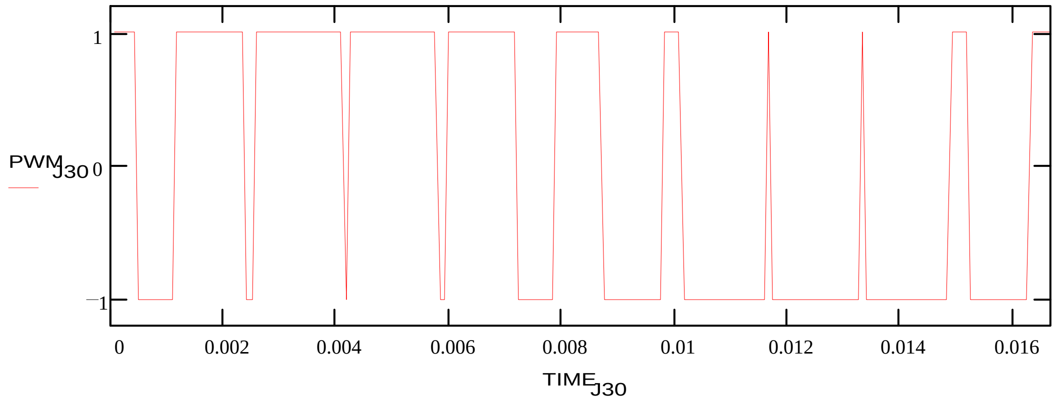

FIGURE #9: PWM WAVEFORM OF CARRIER AND BASE FREQUENCY MODULATED TOGETHER.

In the SCR phase angle controlled circuit, the SCR is gated at some time delay (Td) after the zero cross of the applied voltage. The load voltage is that portion of the sine wave that is let through by the SCR device. The load RMS voltage is varied by varying the time delay before applying the gate signal to the SCR device. In discrete format, the SCR phase angle controlled waveform is evaluated as:

$$Fb(t) := Vp \cdot \sin\left(\frac{2\cdot\pi}{Tb}\cdot t\right)\cdot\left[\sum_{N=0}^{\infty}\left[\mu\left(t-\frac{N\cdot Tb}{2}-Td\right)-\mu\left[t-\frac{(N+1)\cdot Tb}{2}\right]\right]\right]$$

Once again, we will make the base frequency 60 Hz for this modeling example. An example waveform is shown below in figure #10 with a delay angle of 90 degrees and with a peak voltage of 1 and a frequency of 60 Hz.

$J30 := 1..2\cdot N$ $\quad Vp := 1$

$K2 := 20$ $\quad Fi := 60$ $\quad Ti := \frac{1}{Fi}$

$VSIN_{Z1} := 0$ $\quad VSIN_{Z2} := Vp\cdot\sin\left(\pi\cdot\frac{Z2}{N}\right)$ $\quad TIME_{J30} := \frac{J30}{2\cdot Fi\cdot N}$

$VSIN_{Z3} := 0$ $\quad VSIN_{Z4} := Vp\cdot\sin\left(\pi\cdot\frac{Z4}{N}\right)$

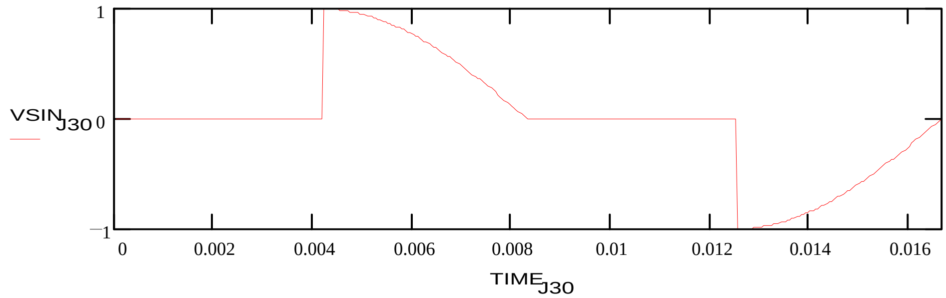

FIGURE #10: SCR GENERATED WAVEFORM

In the transistor controlled chopper circuit, the transistor is used as a switch to turn on and off voltage to the load. The combination of the transistor and load are fed from a DC bus. The load voltage is that portion of the DC bus that is let through by the transistor device. As a side note, the RMS value of voltage applied to the load is controlled by controlling the width of the on state pulse of the transistor. In discrete format, the transistor generated chopper waveform is evaluated as:

$$Fb(t) := Vp\cdot\left[\sum_{N=0}^{\infty} (\mu(t - N\cdot Tb) - \mu(t - Td - N\cdot Tb))\right]$$

Once again, we will make the base frequency 60 Hz for this modeling example and we will make the duty cycle 50%. An example of the waveform is shown below in figure #11.

$DEGREE := 180 \quad VI := 1 \qquad TIME := 2\cdot\pi \quad N := 180$

$Z1 := 1..DEGREE \qquad Z2 := DEGREE + 1, DEGREE + 2..2\cdot N \quad I := 1..2\cdot N$

$VSAMPLE_{Z2} := VI\cdot 0 \qquad VSAMPLE_{Z1} := VI\cdot 1$

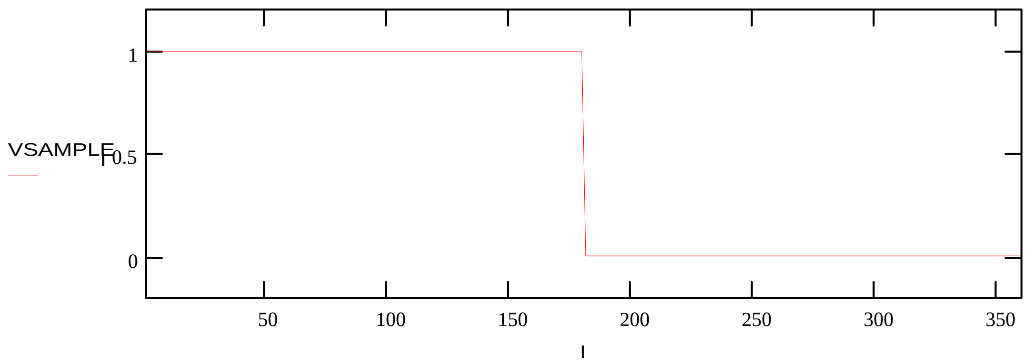

FIGURE #11: CHOPPER GENERATED WAVEFORM

III HARMONIC ANALYSIS OF WAVEFORMS

Now that we have successfully defined our semiconductor device generated waveforms, the next step in our analysis is to calculate the harmonic coefficients and then calculate the total harmonic distortion of these waveforms. Any waveform can be defined as the summation of sinusoidal waveforms with varying coefficients of gain at increasing frequencies. This is the basis for harmonic analysis[3]. It is recommended that the harmonic coefficients be calculated from DC up to at least 10 times the carrier frequency to ensure accuracy for the PWM generated waveform. For the SCR controlled waveform and the chopper waveform, it is recommended that the coefficients be calculated from DC up to at least 15 times base frequency to ensure accuracy for the associated waveform. Only the calculations of the coefficients from DC to 10th harmonic are shown below for brevity. For this analysis, the coefficients from DC to the 100th harmonic were calculated for accuracy.

DEFINITION OF CONSTANTS:

$THETA_{J30} := 2 \cdot \pi \cdot Fi \cdot TIME_{J30}$

$K5 := T1$ $\quad K6 := T1$

DEFINITION OF WAVEFORM UNDER ANALYSIS

$VOUT_{J30} := PWM_{J30}$

DEFINITION OF THE COEFFICIENT FOR THE AVERAGE COMPONENT OF THE WAVEFORM UNDER EVALUATION

$$A_0 := \frac{2}{K5} \cdot \left(\sum_{J30=1}^{K6} VOUT_{J30} \right) \qquad B_0 := 0$$

$A_0 = -0.04$

DEFINITION OF THE COEFFICIENT FOR THE FUNDAMENTAL COMPONENT OF THE WAVEFORM UNDER EVALUATION

$C1 := 1$

$G_{J30} := \cos\left(THETA_{J30} \cdot C1\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C1\right)$

$$A_1 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_1 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_1 = 0 \qquad B_1 = 0.953$

DEFINITION OF THE COEFFICIENT FOR THE SECOND HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C2 := 2$

$G_{J30} := \cos\left(THETA_{J30} \cdot C2\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C2\right)$

$$A_2 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_2 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_2 = -0.02 \qquad B_2 = 0$

DEFINITION OF THE COEFFICIENT FOR THE THIRD HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C3 := 3$

$G_{J30} := \cos\left(THETA_{J30} \cdot C3\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C3\right)$

$$A_3 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_3 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_3 = 0 \qquad B_3 = 0.005$

DEFINITION OF THE COEFFICIENT FOR THE FOURTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C4 := 4$

$G_{J30} := \cos\left(THETA_{J30} \cdot C4\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C4\right)$

$$A_4 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_4 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_4 = -0.005 \qquad B_4 = 0$

DEFINITION OF THE COEFFICIENT FOR THE FIFTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C5 := 5$

$G_{J30} := \cos\left(THETA_{J30} \cdot C5\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C5\right)$

$$A_5 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_5 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_5 = 0 \qquad B_5 = -0.058$

DEFINITION OF THE COEFFICIENT FOR THE SIXTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C6 := 6$

$G_{J30} := \cos\left(THETA_{J30} \cdot C6\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C6\right)$

$$A_6 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_6 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_6 = 0.061 \qquad B_6 = 0$

DEFINITION OF THE COEFFICIENT FOR THE SEVENTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C7 := 7$

$G_{J30} := \cos\left(THETA_{J30} \cdot C7\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C7\right)$

$$A_7 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_7 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_7 = 0 \qquad B_7 = -0.022$

DEFINITION OF THE COEFFICIENT FOR THE EIGHTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C8 := 8$

$G_{J30} := \cos\left(THETA_{J30} \cdot C8\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C8\right)$

$$A_8 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_8 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_8 = 0.283 \qquad B_8 = -1.002 \cdot 10^{-15}$

DEFINITION OF THE COEFFICIENT FOR THE NINTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C9 := 9$

$G_{J30} := \cos\left(THETA_{J30} \cdot C9\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C9\right)$

$$A_9 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_9 := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_9 = 0 \qquad B_9 = -0.017$

DEFINITION OF THE COEFFICIENT FOR THE TENTH HARMONIC OF THE WAVEFORM UNDER EVALUATION

$C10 := 10$

$G_{J30} := \cos\left(THETA_{J30} \cdot C10\right) \qquad H_{J30} := \sin\left(THETA_{J30} \cdot C10\right)$

$$A_{10} := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot G_{J30}\right)\right]\right] \qquad B_{10} := \frac{2}{K5} \cdot \left[\left[\sum_{J30=1}^{K6} \left(VOUT_{J30} \cdot H_{J30}\right)\right]\right]$$

$A_{10} = 0.653 \qquad B_{10} = -2.263 \cdot 10^{-15}$

IV REGENERATION OF WAVEFORM FROM HARMONIC COEFFICIENTS

In order to verify that the majority of harmonic coefficients have been found, it is highly recommended that the waveform be regenerated from the coefficients and compared with the original waveform. If the majority of coefficients have been found, the regenerated waveform should closely approximate the original waveform. (For brevity, only the harmonics from DC to the tenth harmonic level are reconstituted below. In this analysis, the harmonics from DC to the 100^{TH} harmonic were evaluated.) Then the reconstructed waveforms are graphed over the original waveform to verify accuracy. These waveforms are shown in figures #12, #13, and #14.

$$VFB0 := \frac{A_0}{2}$$

$$VFB1_{J30} := A_1 \cdot \cos(THETA_{J30} \cdot C1) + B_1 \cdot \sin(THETA_{J30} \cdot C1)$$

$$VFB2_{J30} := A_2 \cdot \cos(THETA_{J30} \cdot C2) + B_2 \cdot \sin(THETA_{J30} \cdot C2)$$

$$VFB3_{J30} := A_3 \cdot \cos(THETA_{J30} \cdot C3) + B_3 \cdot \sin(THETA_{J30} \cdot C3)$$

$$VFB4_{J30} := A_4 \cdot \cos(THETA_{J30} \cdot C4) + B_4 \cdot \sin(THETA_{J30} \cdot C4)$$

$$VFB5_{J30} := A_5 \cdot \cos(THETA_{J30} \cdot C5) + B_5 \cdot \sin(THETA_{J30} \cdot C5)$$

$$VFB6_{J30} := A_6 \cdot \cos(THETA_{J30} \cdot C6) + B_6 \cdot \sin(THETA_{J30} \cdot C6)$$

$$VFB7_{J30} := A_7 \cdot \cos(THETA_{J30} \cdot C7) + B_7 \cdot \sin(THETA_{J30} \cdot C7)$$

$$VFB8_{J30} := A_8 \cdot \cos(THETA_{J30} \cdot C8) + B_8 \cdot \sin(THETA_{J30} \cdot C8)$$

$$VFB9_{J30} := A_9 \cdot \cos(THETA_{J30} \cdot C9) + B_9 \cdot \sin(THETA_{J30} \cdot C9)$$

$$VFB10_{J30} := A_{10} \cdot \cos(THETA_{J30} \cdot C10) + B_{10} \cdot \sin(THETA_{J30} \cdot C10)$$

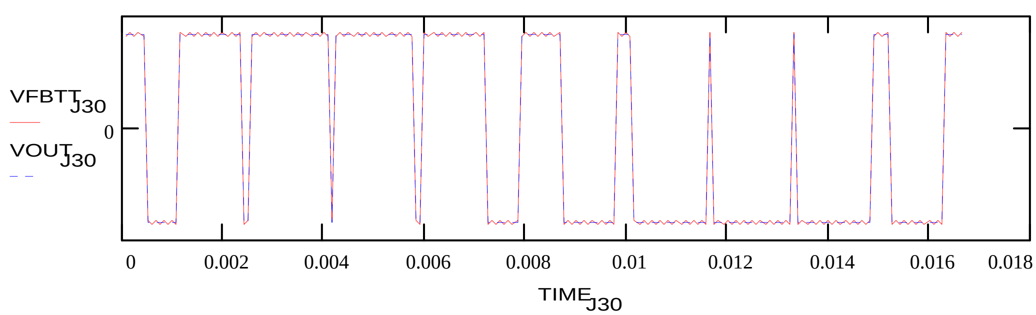

FIGURE #12: COMPARISON OF WAVEFORM REGENERATED FROM HARMONIC COEFFICIENTS (VFBTT) TO THE INITIAL PWM GENERATED WAVEFORM (VOUT)

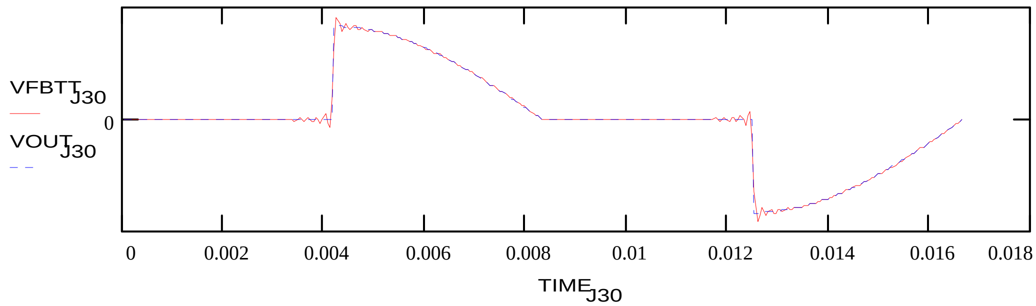

FIGURE #13: COMPARISON OF WAVEFORM REGENERATED FROM HARMONIC COEFFICIENTS (VFBTT) TO THE INITIAL SCR GENERATED WAVEFORM (VOUT)

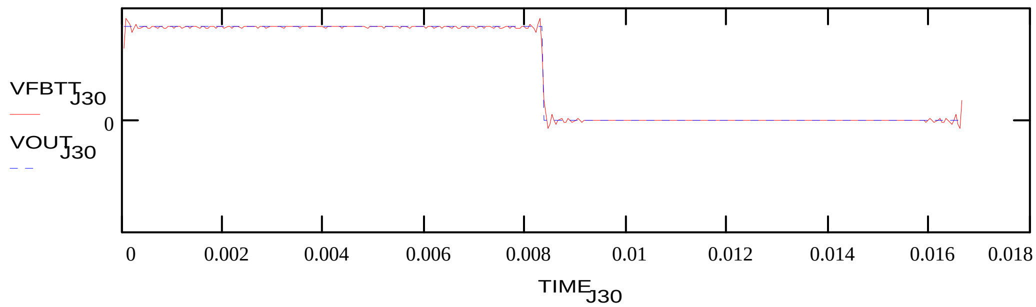

FIGURE #14: COMPARISON OF WAVEFORM REGENERATED FROM HARMONIC COEFFICIENTS (VFBTT) TO THE INITIAL CHOPPER GENERATED WAVEFORM (VOUT)

V CALCULATION OF THE VOLTAGE AND POWER TOTAL HARMONIC DISTORTION VALUES AND DISPLAY OF THE HARMONIC SPECTRUM OF THE SEMICONDUCTOR GENERATED WAVEFORMS.

As shown in figures #12 ,#13, and #14, the regenerated waveforms closely approximate the initial waveforms. This indicates that the majority of significant harmonic coefficients have been calculated. If, in your specific application, the waveforms do not match satisfactorily, then you will need to calculated higher order harmonic coefficients until the waveform match. The harmonic spectrums of these three waveforms from DC to the 100th harmonic frequency are shown in figure #15 through figure #20.

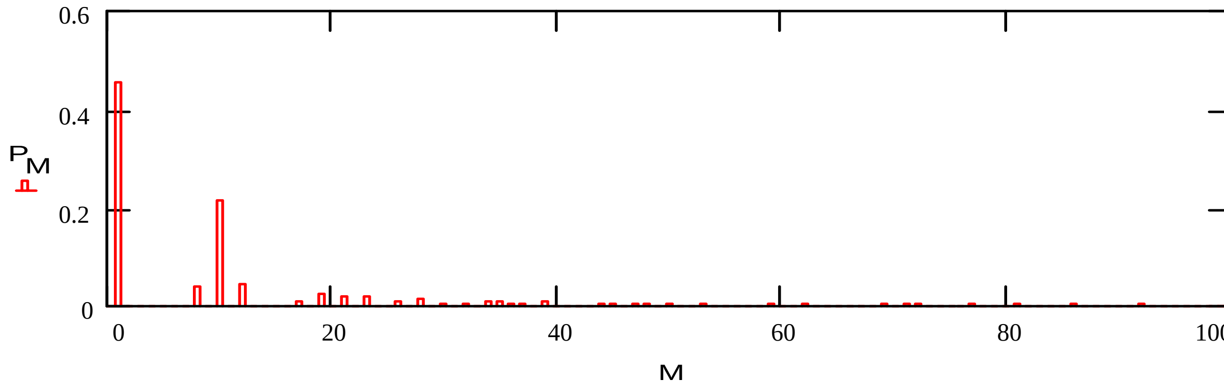

FIGURE # 15: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE PWM GENERATED WAVEFORM..

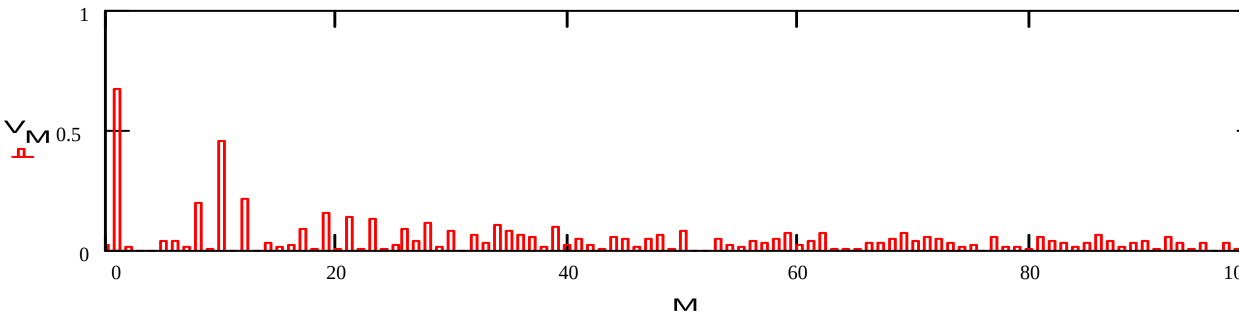

FIGURE # 16: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE PWM GENERATED WAVEFORM.

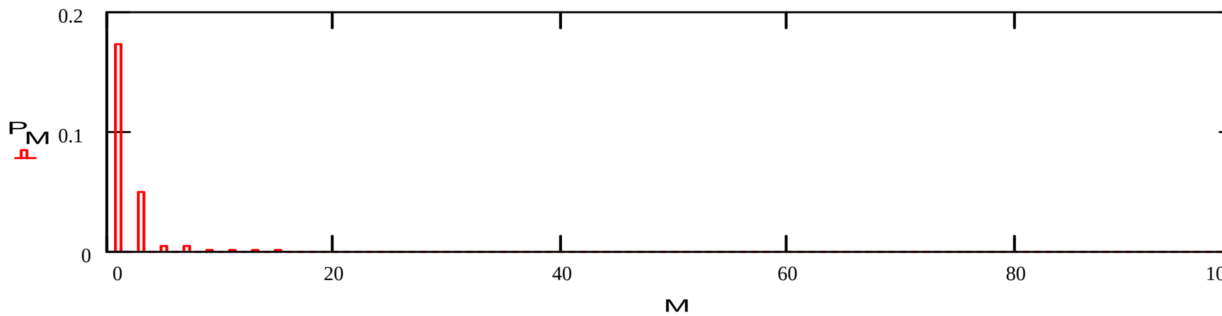

FIGURE # 17: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE SCR GENERATED WAVEFORM..

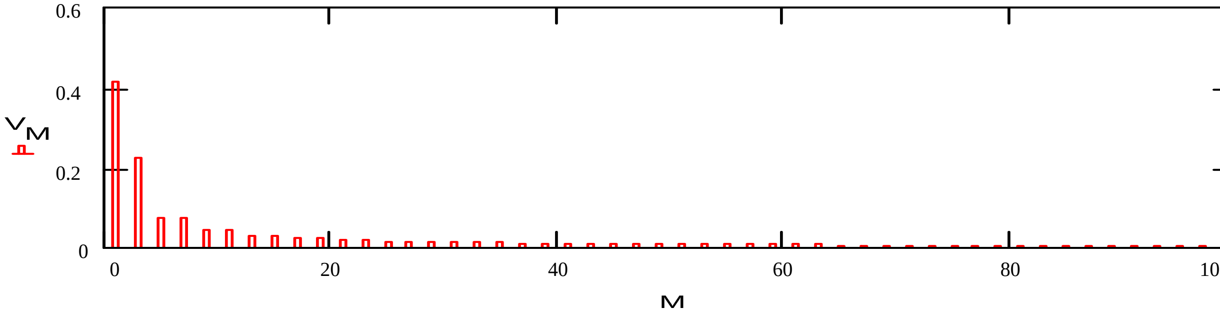

FIGURE # 18: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE SCR GENERATED WAVEFORM.

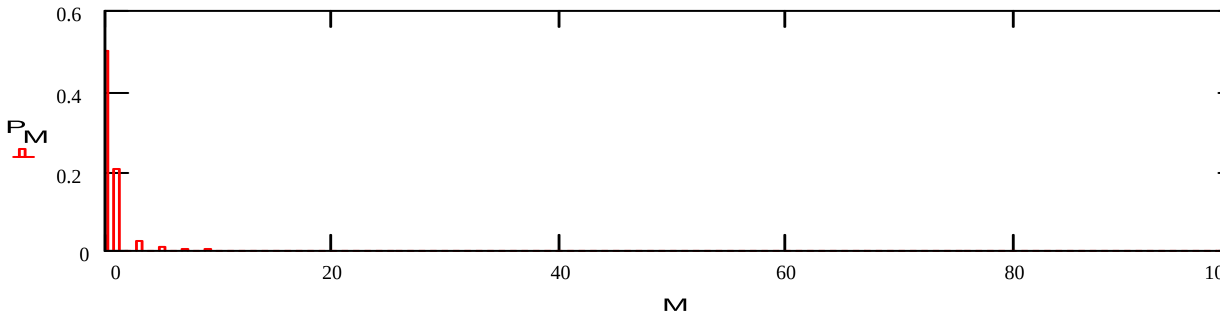

FIGURE # 19: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE CHOPPER GENERATED WAVEFORM..

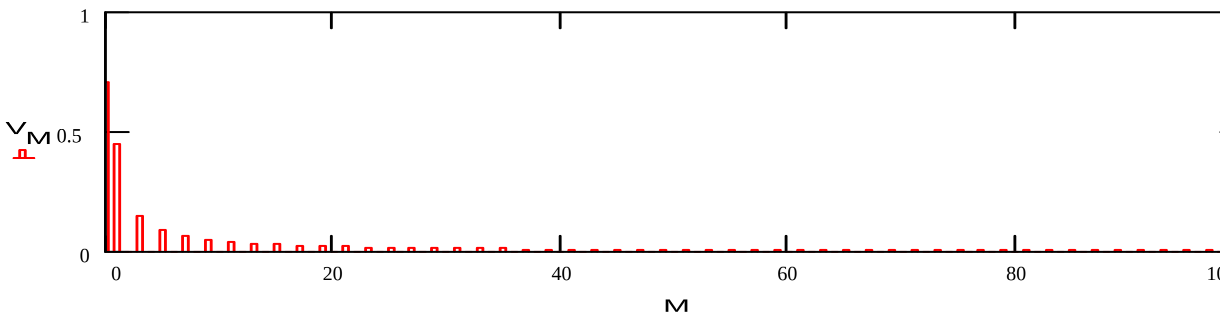

FIGURE # 20: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE CHOPPER GENERATED WAVEFORM.

The total harmonic distortion is defined as the ratio of the RMS value of the harmonic coefficients to the magnitude of the fundamental component[3]. The voltage and power total harmonic distortion levels are calculated below. Note that the power total harmonic distortion assumes that the load is purely resistive. If the load is purely resistive, then the current waveform mirrors the voltage waveform exactly. This allows us to easily calculate the power total harmonic distortion knowing that power = voltage * current. If there are reactive components in the load, then the current waveform does not mirror exactly the voltage waveform and a more detailed analysis is required for power total harmonic distortion.

$Z := 100 \qquad M := 0..Z \qquad M2 := 1..Z$

$$P_M := .5 \cdot \left[(A_M)^2 + (B_M)^2 \right] \qquad V_M := \sqrt{P_M}$$

$$PTHD := \frac{\sum_{M=2}^{Z} P_M}{P_1} \qquad VTHD := \frac{\sqrt{\sum_{M=2}^{Z} (V_M)^2}}{V_1}$$

The voltage total harmonic distortion (VTHD) and power total harmonic distortion (PTHD) values for the PWM generated waveform are shown below:

PTHD = 1.203   VTHD = 1.097

The voltage total harmonic distortion (VTHD) and power total harmonic distortion (PTHD) values for the SCR generated waveform are shown below:

PTHD = 0.426   VTHD = 0.652

The voltage total harmonic distortion (VTHD) and power total harmonic distortion (PTHD) values for the chopper generated waveform are shown below:

PTHD = 0.23   VTHD = 0.48

VI CALCULATION OF LOW PASS FILTER COMPONENT PARAMETERS.

Finally, now that we have thoroughly analyzed our semiconductor generated waveforms, we can define the parameters of a low pass filter required to lower the harmonic level of the output waveform to acceptable levels. The most common low pass filter used for this application is an LC circuit as shown below in figure 21.

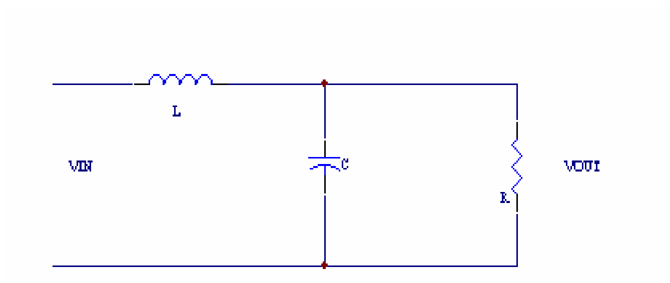

FIGURE #21: BASIC LC FILTER

The transfer function for this filter is defined below:

$$A(S) = (1/LC)/(S^2 + S*(1/RC)+(1/LC))$$

The cut off frequency of this filter is defined below:

$$Fc := \frac{1}{\left(2 \cdot \pi \cdot \sqrt{L \cdot C}\right)}$$

The Q of this filter is defined below:

$$Q := R \cdot \sqrt{\frac{C}{L}}$$

In order to keep the gain of the transfer function maximally flat at the cutoff frequency, we must make Q = 1. Values of Q larger than 1 will generate a gain of larger than 1 for our filter at the cutoff frequency. While we have chosen our cutoff frequency at a point where there is very little harmonic content from our waveform (at the second harmonic), it is good practice in industrial applications to ensure that the gain of the filter is 1 or less throughout the range of the harmonic spectrum. This is due to the possibility of an external source of harmonics located somewhere else in the power circuit that may be generating harmonics at our cutoff frequency. If this is the case, and we do not carefully choose our filter parameters, we may magnify the unwanted harmonics generated from the external source. We can define the equivalent load resistance (R) for our single phase equivalent circuit as the line to neutral voltage divided by the rated current of the application. For our application, we will let the maximum current be 10 amps and the line to neutral voltage to be 277 volts. The transfer function for our application is defined below and the Bode plot [3] of the transfer function is shown in figure #22.

$Vln := 277$ $Fc := 120$ $Q := 1$ $IL := 10$ $R := \frac{Vln}{IL}$ $R = 27.7$

$$L := \frac{R}{Q \cdot 2 \cdot \pi \cdot Fc} \qquad L = 0.037$$

$$C := \frac{Q}{2 \cdot \pi \cdot Fc \cdot R} \qquad C = 4.788 \cdot 10^{-5}$$

$$GAIN_{M2} := \left| \frac{\frac{1}{L \cdot C}}{\left[\frac{1}{L \cdot C} - (2 \cdot \pi \cdot Fi \cdot M2)^2\right] + IM \cdot \frac{2 \cdot \pi \cdot Fi \cdot M2}{(R \cdot C)}} \right|$$

$$DBA_{M2} := 10 \cdot \log\left(GAIN_{M2}\right)$$

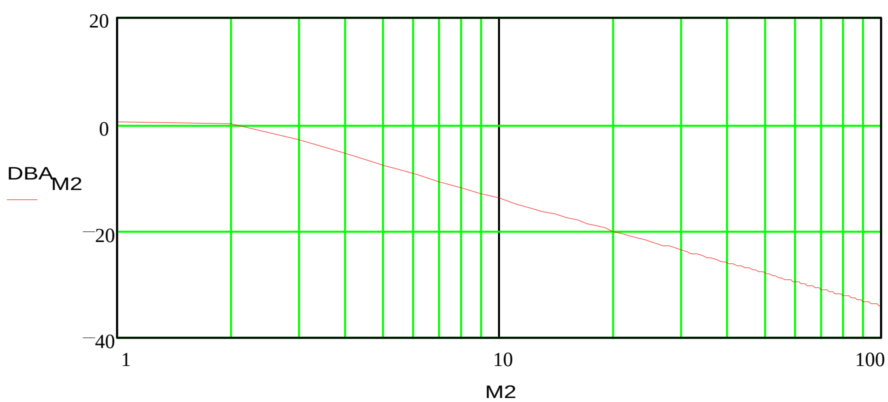

FIGURE #22: TRANSFER FUNCTION OF THE PROPOSED FILTER.

VII CALCULATION OF FILTER OUTPUT WAVEFORM

Now to determine the output waveform of the filter, we multiply the harmonic coefficients calculated above by the gain of our transfer function at each of the harmonic frequencies to obtain new harmonic coefficients. Then we reconstruct the filter output waveform using the new harmonic coefficients. Again, for simplicity, only the first 10 coefficient calculations are shown.

$AN_0 := A_0 \qquad BN_0 := B_0$

$AN_{M2} := GAIN_{M2} \cdot A_{M2} \qquad BN_{M2} := GAIN_{M2} \cdot B_{M2}$

$$VFB0N := \frac{AN_0}{2}$$

$$VFB1N_{J30} := AN_1 \cdot \cos(THETA_{J30} \cdot C1) + BN_1 \cdot \sin(THETA_{J30} \cdot C1)$$

$$VFB2N_{J30} := AN_2 \cdot \cos(THETA_{J30} \cdot C2) + BN_2 \cdot \sin(THETA_{J30} \cdot C2)$$

$$VFB3N_{J30} := AN_3 \cdot \cos(THETA_{J30} \cdot C3) + BN_3 \cdot \sin(THETA_{J30} \cdot C3)$$

$$VFB4N_{J30} := AN_4 \cdot \cos(THETA_{J30} \cdot C4) + BN_4 \cdot \sin(THETA_{J30} \cdot C4)$$

$$VFB5N_{J30} := AN_5 \cdot \cos(THETA_{J30} \cdot C5) + BN_5 \cdot \sin(THETA_{J30} \cdot C5)$$

$$VFB6N_{J30} := AN_6 \cdot \cos(THETA_{J30} \cdot C6) + BN_6 \cdot \sin(THETA_{J30} \cdot C6)$$

$$VFB7N_{J30} := AN_7 \cdot \cos(THETA_{J30} \cdot C7) + BN_7 \cdot \sin(THETA_{J30} \cdot C7)$$

$$VFB8N_{J30} := AN_8 \cdot \cos(THETA_{J30} \cdot C8) + BN_8 \cdot \sin(THETA_{J30} \cdot C8)$$

$$VFB9N_{J30} := AN_9 \cdot \cos(THETA_{J30} \cdot C9) + BN_9 \cdot \sin(THETA_{J30} \cdot C9)$$

$$VFB10N_{J30} := AN_{10} \cdot \cos(THETA_{J30} \cdot C10) + BN_{10} \cdot \sin(THETA_{J30} \cdot C10)$$

The outputs of the low pass filter for the three modeled waveforms are shown below in figures #23, #24, and #25. As can be plainly seen, these waveforms much more closely represent a sine wave then the original waveforms. This is due to the reduction of harmonic frequencies from our original waveforms by the low pass filter.

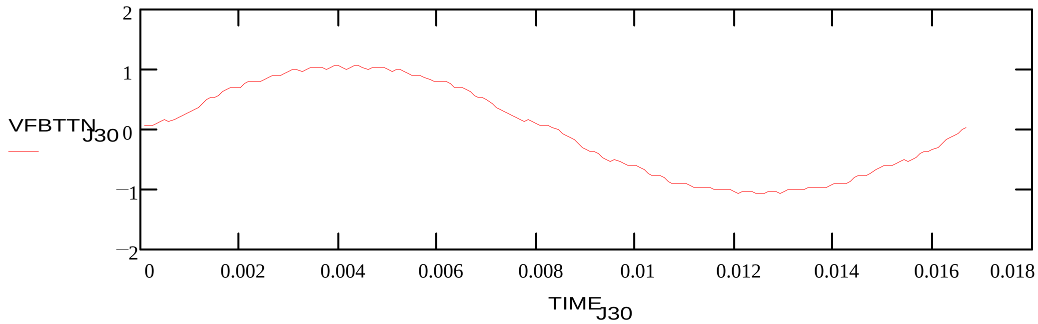

FIGURE #23: PWM GENERATED VOLTAGE WAVEFORM OUT OF FILTER AT MAXIMUM CURRENT DRAW.

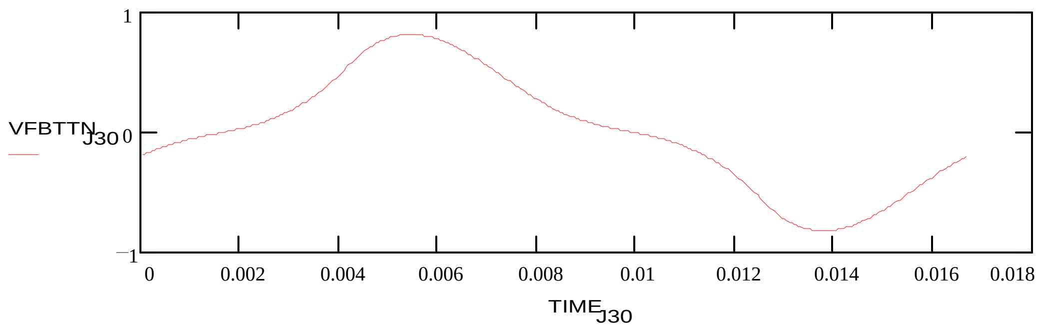

FIGURE #24: SCR GENERATED VOLTAGE WAVEFORM OUT OF FILTER AT MAXIMUM CURRENT DRAW.

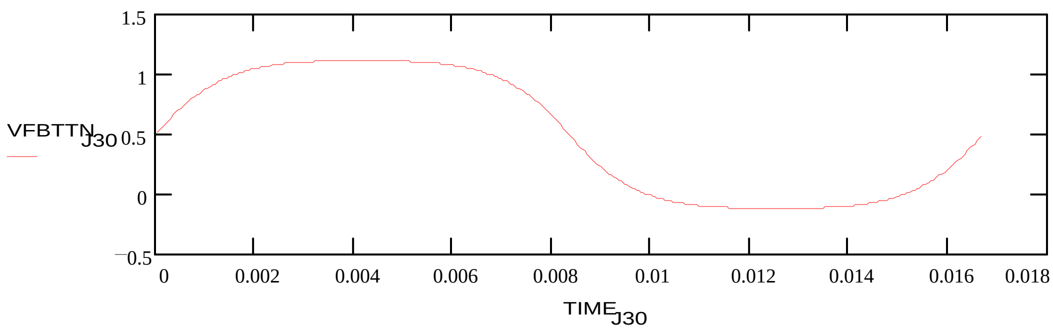

FIGURE #25: CHOPPER GENERATED VOLTAGE WAVEFORM OUT OF FILTER AT MAXIMUM CURRENT DRAW.

VIII CALCULATION OF THE VOLTAGE AND POWER TOTAL HARMONIC DISTORTION VALUES AND DISPLAY OF THE HARMONIC SPECTRUM OF THE SEMICONDUCTOR GENERATED WAVEFORMS AT THE OUTPUT OF THE LOW PASS FILTER.

We can now calculate the total harmonic voltage and power distortion levels of these waveforms as was done for the initial waveforms above as well as the voltage and power spectrum of the three waveforms from DC to the 100^{TH} harmonic. The results are shown below in figure #26 through figure #31:

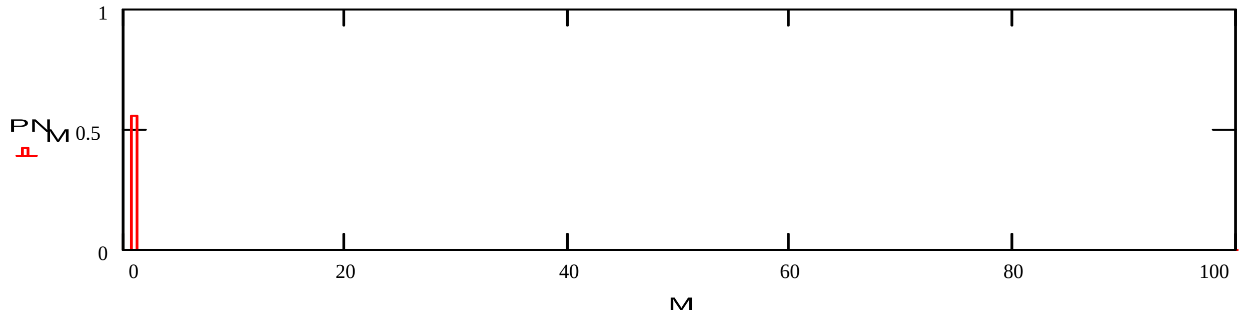

FIGURE # 26: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE PWM GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

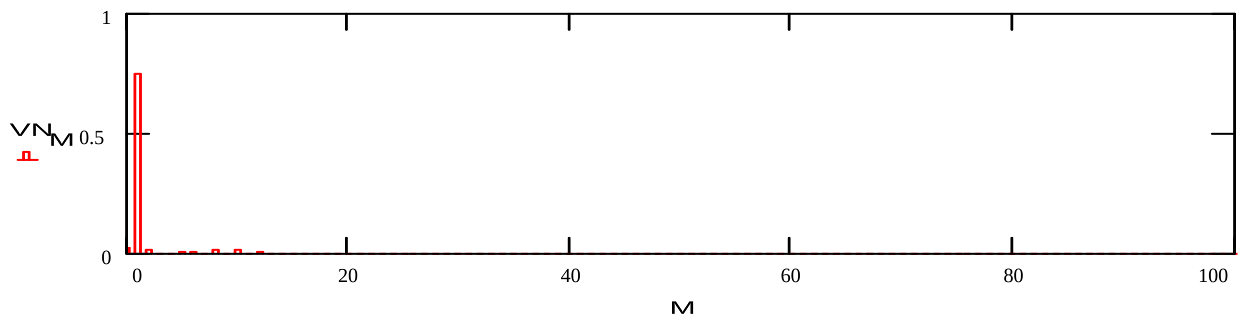

FIGURE # 27: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE PWM GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

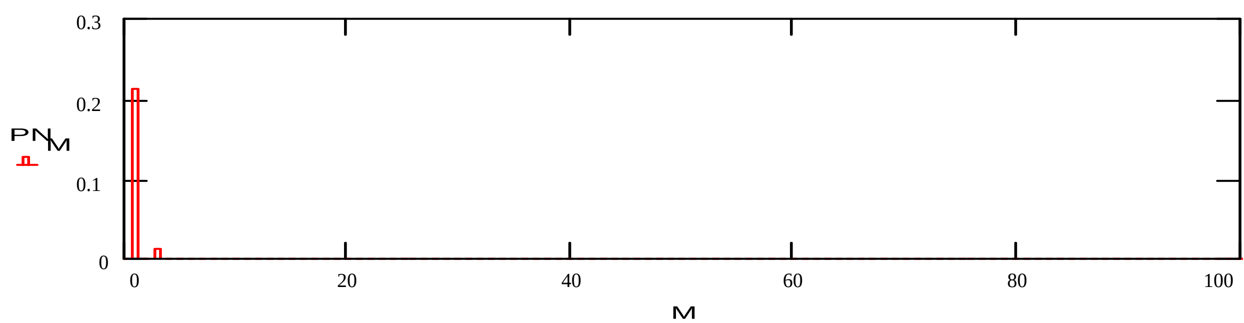

FIGURE # 28: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE SCR GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

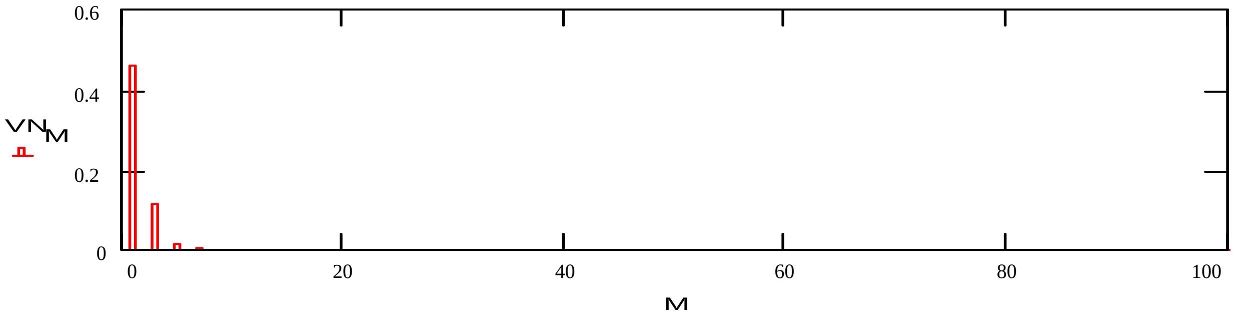

FIGURE # 29: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE SCR GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

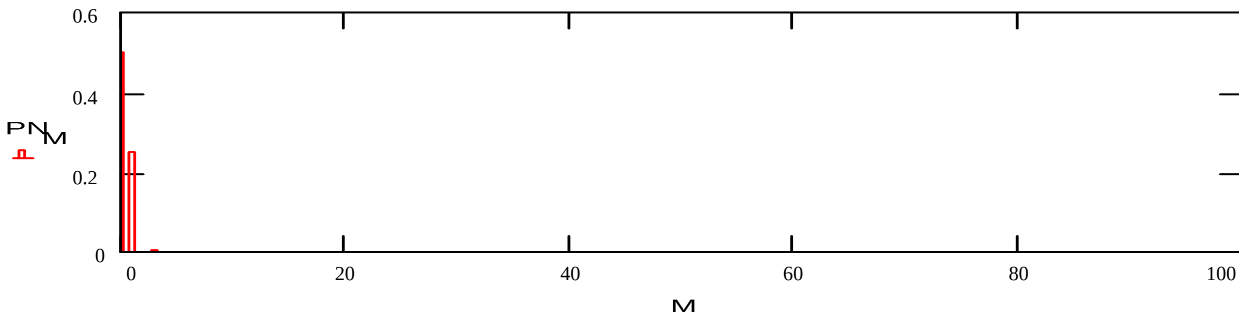

FIGURE # 30: GRAPHICAL REPRESENTATION OF POWER SPECTRUM FOR THE CHOPPER GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

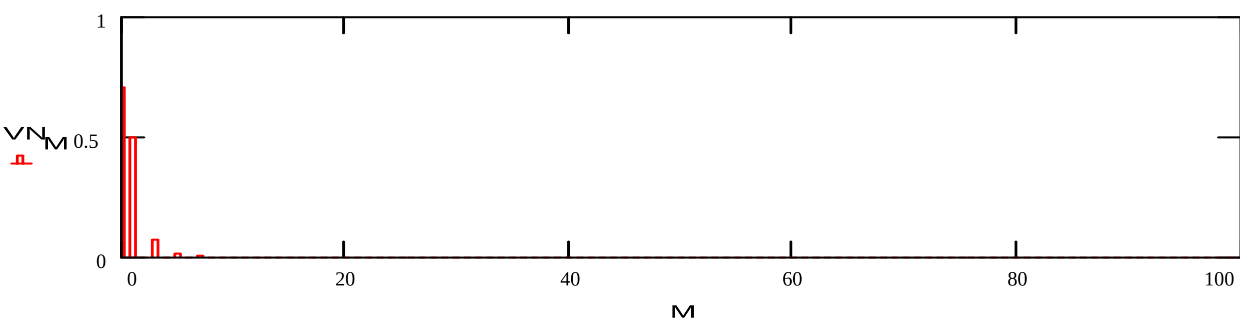

FIGURE # 31: GRAPHICAL REPRESENTATION OF VOLTAGE SPECTRUM FOR THE CHOPPER GENERATED WAVEFORM AS SEEN AT THE OUTPUT OF THE FILTER.

The voltage total harmonic distortion (VTHDN) and power total harmonic distortion (PTHDN) values for the PWM generated waveform are shown below:

PTHD = 0.002   VTHD = 0.039

The voltage total harmonic distortion (VTHDN) and power total harmonic distortion (PTHDN) values for the SCR generated waveform are shown below:

PTHD = 0.063   VTHD = 0.252

The voltage total harmonic distortion (VTHDN) and power total harmonic distortion (PTHDN) values for the chopper generated waveform are shown below:

PTHD = 0.025   VTHD = 0.157

IX CONCLUSION

We have demonstrated a method of mathematically modeling three of the most common semiconductor generated waveforms. Then we have analyzed these waveforms for harmonic content. Finally, we have shown a method of developing a low pass filter to reduce the harmonic content of these waveforms and then regenerated the filtered waveforms from harmonic content. For the PWM generated waveform, the voltage total harmonic distortion level was reduced from 109.7% to 3.9% and the power total harmonic distortion level was reduced from 120.3% to 0.2%. For the SCR generated waveform, the voltage total harmonic distortion level was reduced from 65.2% to 25.2% and the power total harmonic distortion level was reduced from 42.6% to 6.3%. For the chopper generated waveform, the voltage total harmonic distortion level was reduced from 48% to 15.7% and the power total harmonic distortion level was reduced from 23% to 2.5%.

It is interesting to note that, with no filtering, the PWM waveform generates the highest magnitude of total voltage harmonic distortion, yet with the low pass filter attached, the PWM waveform generates the lowest magnitude of total voltage harmonic distortion. This is due to the fact that the significant harmonic coefficients in the PWM generated waveform are of orders of frequency much higher than in the SCR phase angle generated waveform or the chopper generated waveform. At these higher values of frequency, the attenuation of the low pass filter is greater. Therefor, the same low pass filter will do a better job of reducing harmonics generated from the PWM generated waveform than it will do in the other two waveforms under consideration.

X REFERENCES

[1] Bose, B. K.., "MODERN POWER ELECTRONICS Evolution, Technology, and Applications", IEEE Press, 1992.

[2] Stevenson, William D. Jr., "Elements of Power System Analysis", McGraw Hill, 1982.

[3] Jordan, Edward C., "Reference Data for Engineers: Radio, Electronics, Computer, and Communications",

Howard W. Sams & Company, 1985